

**TIME DOMAIN MODELING & POWER OUTPUT FOR A HEAVING POINT  
 ABSORBER WAVE ENERGY CONVERTER**

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**NOMENCLATURE**

$\omega$	Frequency domain
$t$	Time domain
$F_{ex}$ and $f_{ex}$	Excitation force
$F_{rad}$ and $f_{rad}$	Radiation force
$F_{res}$	Frequency-domain added mass
$\eta(t)$	Water surface elevation
$F_{damp}$	Damping force
$F_{tun}$	Tuning force

**ABSTRACT**

This paper presents, assesses, and optimizes a point absorber wave energy converter (WEC) through numerical modeling, simulation, and analysis in time domain. Wave energy conversion is a technology especially suited for assisting in power generation in the offshore oil and gas platforms. A linear frequency domain model is created to predict the behavior of the heaving point absorber WEC system. The hydrodynamic parameters are obtained with AQWA, a software package based on boundary element methods. A linear external damping coefficient is applied to enable power absorption and an external spring force is introduced to tune the point absorber to the incoming wave conditions. The external damping coefficient and external spring forces are the control parameters, which need to be optimized to maximize the power absorption. Two buoy shapes are tested and a variety of diameters and drafts are compared. Optimal shape, draft, and diameter of the model are then determined to maximize its power absorption capacity. Based on the results generated from the frequency domain analysis, a time domain analysis was also

conducted to derive the responses of the WEC in the hydrodynamic time response domain. The time domain analysis results allowed us to estimate the power output of this WEC system.

**Keywords:** wave energy converter, point absorber, numerical modeling, time domain analysis, CFD

**1 INTRODUCTION**

In most applications in order to get a better understanding of a Wave Energy Converter (WEC) an analysis and model are generated in the frequency domain. However, the use of a time domain model in some cases can be extremely useful. Instances of this occur when non-linear effects need to be included when it becomes necessary to observe the reactions of a WEC in a time series. The best approach to this up to date is the use of Computational Fluid Dynamics (CFD) models, which is based on approximations of the Navier-Stokes equations. For the study done in this paper the CFD program ANSYS was used to generate the responses of the WEC in the hydrodynamic time response domain [1]. These results were derived from the frequency domain results found in a previous study [2]. Once these results in the time domain were calculated, an estimation of what the power output may be was formulated.

**2 FREQUENCY DOMAIN SOLUTION**

**2.1 Concept and the Equation of Motion**

In a previous study [2] a concept for a WEC was modeled in the frequency domain to ascertain its capacity for power absorption. Power absorption refers to the amount of power a

WEC can generate in irregular long-crested waves. This concept was based off of a fixed, heaving point absorbing WEC with a direct mechanical power take-off (PTO) system. The point absorbing WEC is a simple technology consisting of buoys or floating bodies that capture the wave's heaving motion. In a point absorbing system, buoys move through a single degree of motion with the ocean waves as depicted by the letter z in figure. This concept was designed to be fixed on to off-shore oil and gas platforms to help supplement their power needs. A simple and robust design was proposed for the demanding operating environment.

Using linear theory, the equation of motion of a floating body, oscillating in heave mode is written as:

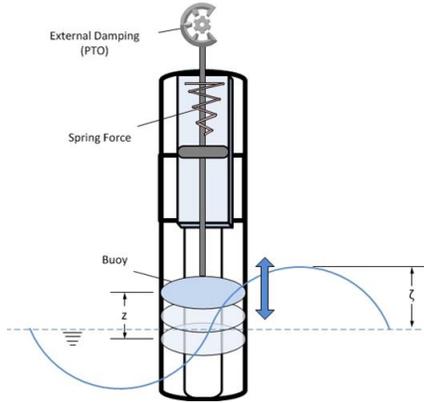


Fig.1 Schematic representation of a fixed heaving point absorber with applied spring control

$$m \frac{d^2 z}{dt^2} = F_{ex} + F_{rad} + F_{res} + F_{damp} + F_{run} \quad (1)$$

The PTO forces associated with this system are the damping force ( $F_{damp}$ ) which is caused by the force of the generator on the floating body, and the tuning force ( $F_{run}$ ) caused by the spring forces used to tune the system. These forces are assumed to be linear, therefore allowing the floating body to respond to the harmonic excitation forces ( $F_{ex}$ ,  $F_{rad}$ ,  $F_{res}$ ) caused by the wave. This gives our equation an analytical solution. The excitation force as well as the radiation force are the primary forces that define Impulse Response Functions (IRFs). These two forces are defined by:

$$F_{ex}(t) = \int_{-\infty}^{\infty} \eta(\tau) f_{ex}(t - \tau) d\tau \quad (2)$$

$$F_{rad}(t) = \int_{-\infty}^t f_{rad}(t - \tau) \dot{x}(\tau) d\tau \quad (3)$$

where  $F_{ex}(t)$  is found by the convolution of the water surface elevation,  $\eta(t)$  with the non-casual IRF,  $f_{ex}(t)$ . Secondly,  $F_{rad}(t)$  is caused by radiating waves and is determined by the convolution of the radiation IRF,  $f_{rad}(t)$ , with the WEC's velocity,  $\dot{x}$ .

Next, the complex frequency-domain excitation force,  $f_{ex}(i\omega)$ , is used to calculate the time-domain excitation IRF,  $f_{ex}(t)$ , the frequency domain radiation  $f_{rad}(\omega)$  is used to calculate

the time-domain radiation IRF,  $f_{rad}(t)$ , and the limit at infinity of the frequency-domain added mass is evaluated,  $F_{res}(\infty)$ . These hydrodynamic terms,  $f_{ex}(t)$ ,  $f_{rad}(t)$  and  $F_{res}(\infty)$ , are the building blocks of the time-domain WEC Equations of Motion (EOM). These time-domain EOM use the IRF formulation and were first introduced by Cummins for ship motions in 1962 [7].

$$f_{ex}(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} f_{ex}(i\omega) e^{i\omega t} d\omega \quad (4)$$

$$f_{rad}(t) = \frac{2}{\pi} \int_0^{\infty} f_{rad}(\omega) \cos(\omega t) d\omega \quad (5)$$

## 2.2 Hydrodynamic Parameters of the Frequency Domain Results

In order to understand the WEC's response in the time domain an analysis in the frequency domain must be established. This analysis produces the hydrodynamic parameters which are used to formulate how a WEC will react over a given time period. The data collected from this formulation can then be used to determine the amount of power produced from the PTO system. The flowchart in figure 2 illustrates how the model as a whole works together to define the power output of the system.

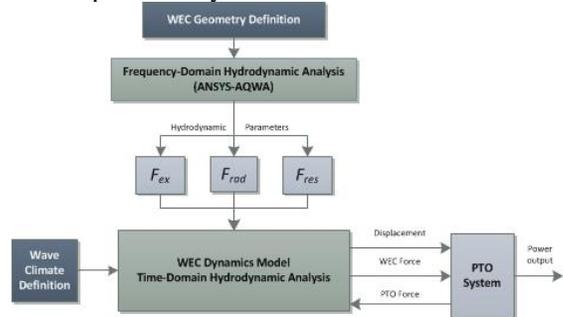


Fig.2 Flowchart of a WEC dynamic model

The results for the frequency-domain analysis are shown graphically in figures 3 through 5. Again, these results were obtained in the previous study [2]. These results will serve as a basis for the time-domain analysis.

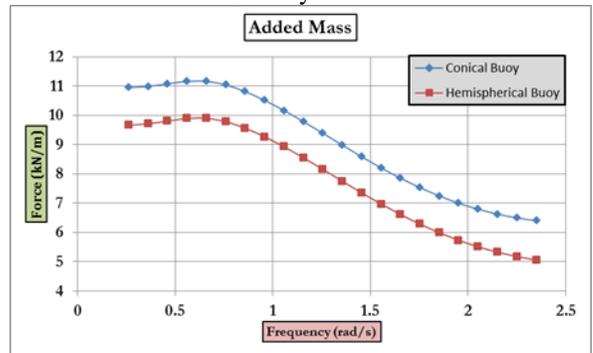


Fig. 3 Added mass of a floating body with a diameter of 3.5 meters and a draft of 2 meters

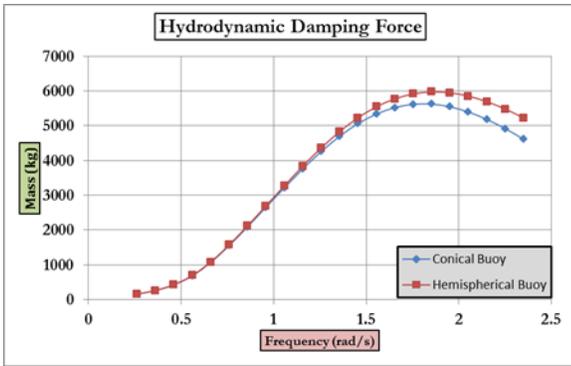


Fig. 4 Radiation damping of a floating body with a diameter of 3.5 meters and a draft of 2 meters

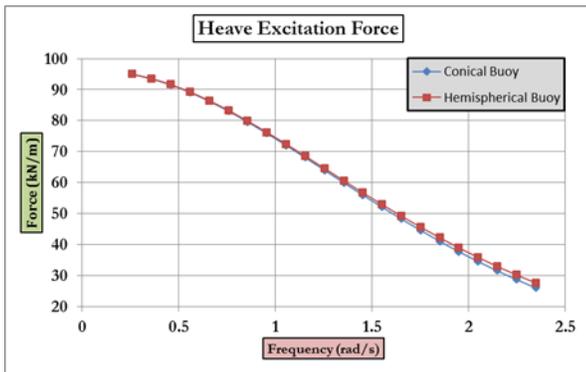


Fig. 5 Heave excitation force of a floating body with a diameter of 3.5 meters and a draft of 2 meters

### 3 OPTIMUM SIZE AND INPUT PARAMETERS

Once the frequency modeling was complete then an optimum size for the buoy can be decided. In the previous study a conical shaped point absorber with diameter of 3.5 meters and a waterline draft of 2 meters was found to be the optimal size and shape for the buoy [2]. Figure 6 gives a CAD drawing of the buoy detailing the dimensions and waterline reference.

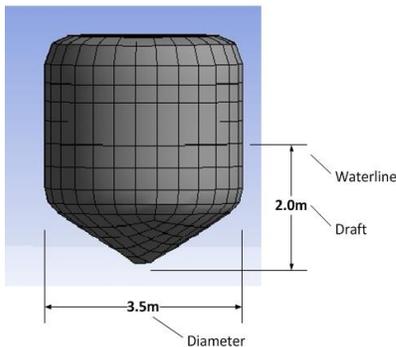


Fig. 6 CAD drawing of a conical shaped buoy with a diameter of 3.5 meters and a waterline draft of 2.0 meters

The input parameters needed to run AQWA's hydrodynamic time response rely on results from the frequency domain analysis and the type of wave climate. From the frequency domain analysis we get the radiation impulse response function

and the exciting force transfer function, which are obtained from AQWA. The results of this analysis are listed in section II-B. The wave climate used for the analysis consisted of an irregular wave input. This irregular wave consisted of a Jonswap type wave, a significant wave height of 1.1 meters, and a peak wave frequency of 1.1 rad/s. The frequency range used was between 0.638 and 5.621 rad/s.

### 4 AN APPLICATION EXAMPLE

The response amplitude operator (RAO) of the buoy position is defined as the ratio between the displacement amplitude of the uncontrolled buoy, responding to a harmonic excitation, and the incident wave amplitude [3]. The RAO can be calculated directly with AQWA [1]. It has been computed with the time domain model from the steady state response of the buoy to a regular incident wave. A window of 20 seconds was used to visualize the buoys reaction. The number of steps used was 120 giving us a time step of 0.168 seconds. Results from this analysis consisted of the total force and actual position of the buoy over the 20 second period.

In figure 7 the resulting buoy forces acting on the system are shown over time. The maximum force acting in the positive heave direction is 6.9 kN and the maximum force in the negative heave direction of 5.5 kN. This creates an average force on the system of 4.1 kN.

Figure 8 shows the position of the buoy with respect to time. The maximum value for the buoy in the positive heave direction is 0.83 meters and the minimum value is based off of datum of 0 meters. Based on the graph the buoy holds an average speed 0.75 m/s over the course 20 seconds.

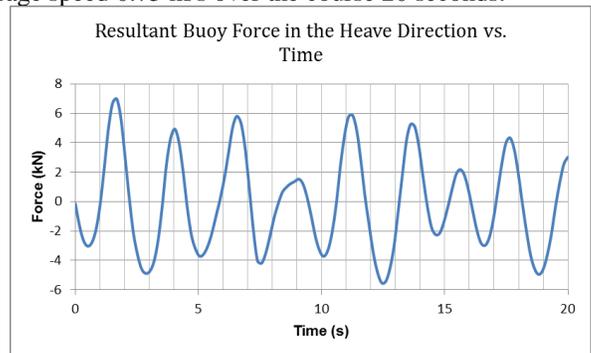


Fig. 7 Resulting forces of a heaving buoy over a period of 20 seconds

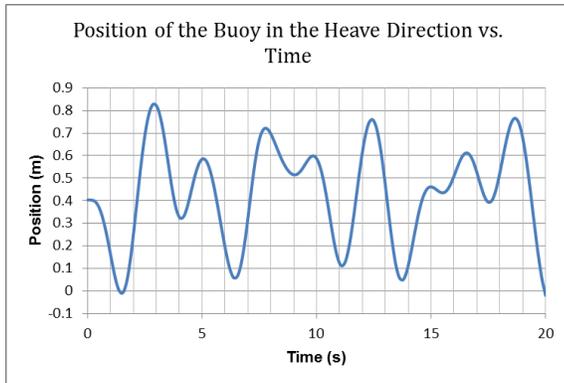


Fig. 10 Displacement of a heaving buoy over a period of 20 seconds

## 5 CONCLUSIONS

A time domain model has been implemented in AQWA. The equation of motion of the heaving point absorber is described by the sum of all the forces acting on a floating body. This is defined by the hydrodynamic forces caused by the oscillating ocean wave and the external forces caused by the power generation system. The time domain model (equations 4 and 5) uses input from AQWA's frequency domain analysis for the hydrodynamic parameters (infinite frequency limit of the added mass, exciting force and radiation impulse response function). When the results are applied to a selected PTO system a power output of 150 kW is reached for a single floating body and 1.2 MW for an array.

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